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PAPER

Conjecture of new inequalities for some selected thermophysical properties values

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Abstract

In 2005 it was rigorously shown with string theory methods that there is a lower bound for the ratio of the shear viscosity η and the volume density of entropy $s = S/V$ given by $\eta/s \geqslant \hbar/(4\pi k_B)$. Here we extend this result in a heuristic manner to other ratios of thermophysical properties. We conjectured that there are rigorous non-zero lower bounds for the Lorenz number L as well as other combinations of equilibrium and transport properties. We suggest that the lower bounds and the corresponding inequalities can be written in terms of the Planck units. We show that some of the proposed new inequalities set severe constraints on the behavior and properties of ordinary matter.

1. Introduction

In 2005 Kovtun, Son, and Starinets (KSS) [1] studied the shear viscosity in strongly interacting quantum field theories. These studies are of considerable importance in black hole and high-energy physics for which one example is the quark-gluon plasma produced in heavy-ion collisions, see e.g. [2, 3]. In particular KSS concentrate on the ratio $\kappa = \eta/s$ of the shear viscosity η to the volume density of entropy $s = S/V$. They conjectured that $\kappa_0 = \hbar/(4\pi k_B) \approx 6.08 \cdot 10^{-13}$ Ks is a lower bound to κ for all single-component nonrelativistic systems of particles with spin 0 or 1/2. This leads to the universal inequality $\kappa \geqslant \kappa_0$.

$$\kappa = \frac{\eta}{s} \geqslant \kappa_0 = \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13}\ \mathrm{K\ s} \tag{1}$$

κ cannot become smaller than κ_0 which sets a rigorous lower bound on this ratio. Since its discovery numerous papers in high-energy and black-hole physics have concentrated on this inequality, see e.g. [4–10]. Because entropy and shear viscosity are common thermophysical properties with a clear definition for ordinary matter, this inequality has been tested with experimental data for ordinary fluids like the rare gases, H_2, N_2, CH_4, and CF_4 [11]. For all systems studied so far it has been shown that $\kappa > \kappa_0$ holds well. Interestingly, the minimum value κ_{min} of κ was observed in the vicinity of the critical point of ordinary systems [11], of high-energy matter [12] and also of high-temperature superconductors [13]. It was observed for ordinary fluids [11] that the minimum value κ_{min} of κ of the rare gases and other small molecules is in the range between $\kappa_{min} \approx (9 \ldots 100)$ κ_0. Roughly speaking κ_{min} is in the same order of magnitude as κ_0. This result is not necessarily expected for these fluids because the lower bound κ_0 is obtained from theoretical considerations of very high-energy physical phenomena including black holes using string theory methods [1].

There are much more thermophysical properties than η and s. The question is whether there are more combinations of other thermophysical properties which are bound from below. This question will be addressed in a heuristic manner. Existing inequalities are summarized in table 1. We show that their accepted lower bounds can be recovered with our conjecture. New inequalities are presented in sections 3.2 and 3.3, whereas in sections 3.1 and 3.4 special conclusions are drawn for properties, which have the dimension *time times temperature*.

Table 1. Uncertainty relations (left). Note that the numerical factors on the rhs of the inequalities are in some cases subject of debate.

Complementary pair	Reference	lhs (first column) in terms of Planck units, table 2
(1) $\Delta p \Delta x \geqslant \hbar/2$	Heisenberg [14]	$m_P c \ell_P = \hbar$
(2) $\Delta E \Delta t \geqslant \hbar/2$	Heisenberg [14]	$E_P t_P = \hbar$
(3) $\Delta J \Delta \theta > \hbar$	Heisenberg [14], Aharonov and Reznik [28]	$\hbar \cdot 1 = \hbar$
(4) $\frac{1}{T^2}\Delta E \Delta T \geqslant k_B$	various, see [17–22, 25–27]	$\frac{1}{T_P^2} E_P T_P = k_B$
(5) $\Delta E \Delta \frac{1}{T} \geqslant k_B$	various, see [17, 19, 21, 22, 25–27, 40]	$E_P \frac{1}{T_P} = k_B$
(6) $\Delta F \Delta \frac{1}{T} \geqslant k_B/2$	Zimmermann [33]	$E_P \frac{1}{T_P} = k_B$
(7) $\Delta \frac{\Pi}{T} \Delta V \geqslant k_B$	Schlögl [22]	$(F_P/\ell_P^2)\ell_P^3/T_P = k_B$
(8) $\Delta \frac{\mu}{T} \Delta N \geqslant k_B$	Schlögl [22]	$(E_P/T_P) \cdot 1 = k_B$
(9) $\Delta \dot{S} \Delta t \geqslant k_B/2$	Zimmermann [31–33]	$(S_P/t_P) t_P = k_B$
(10) $\Delta m \Delta t > \hbar/c^2$	Landau and Peierls [16], Aharonov and Reznik [28]	$m_P t_P = \hbar/c^2$
(11) $\Delta T \Delta t \geqslant \hbar/k_B$	de Sabbata and Sivaram [23, 24], Gillies and Allison [29, 30]	$T_P t_P = \hbar/k_B$
(12) $\Delta T \Delta x \geqslant \hbar c/(4\pi k_B)$	Viaggiu [39]	$\hbar c/k_B$
(13) $\Delta p \Delta t \geqslant \beta \hbar/c$	Landau and Peierls [16] ($\beta = 1$), Dodonov and Dodonov [34] ($\beta = 1/2$)	$m_P c t_P = \hbar/c$
(14) $\Delta x \Delta t \geqslant G\hbar/c^4$	Burderi *et al* [38]	$\ell_P t_P = G\hbar/c^4$

2. Methodology

2.1. Uncertainty relations

We start with the Heisenberg uncertainty principle (HUP) originally formulated as $p_1 x_1 \sim h$ [14]. Nowadays the HUP is almost exclusively presented as $\Delta p \Delta x \geqslant \hbar/2$. Here p and x are the momentum and position, respectively, and $h = 2\pi\hbar$ is Planck's constant. Δ is some kind of inherent uncertainty or indeterminacy. Beside its statistical interpretation $\Delta A = (\langle A^2 \rangle - \langle A \rangle^2)^{1/2}$ for any observable A it can also be related to the uncertainty of measurement [15]. The Heisenberg uncertainty principle sets a rigorous and very fundamental limit on the product $\Delta p \Delta x$. It is one of the cornerstones of quantum mechanics.

During the last years many other uncertainty relations of the type $\Delta A \Delta B \geqslant C$ have appeared in the literature [14, 16–40], see table 1. In principle A and B are experimentally accessible properties which form a complementary pair. Examples for A, B, and C are energy E and time t with $C = \hbar/2$, position x and time t with $C = G\hbar/c^4$, but also temperature T and length x with $C = \hbar c/k_B$. It might be interesting to note that also thermodynamic properties like temperature T, Helmholtz energy F, chemical potential μ, volume V, and pressure Π do occur in these formulations. Indeed, the role of thermodynamic uncertainty relations as fundamental bounds in biological and chemical physics are currently under investigation [35–37]. It turns out that the bound $C > 0$ can always be expressed in terms of the fundamental physical constants. It should be stressed that the inequalities given in table 1 are obtained from sophisticated gedanken experiments as well as elaborate and sometimes lengthy treatments of the underlying statistical, quantum, thermodynamic, or gravitational theories.

2.2. Generalized uncertainty principle

We are now looking for a simple merely heuristic way to obtain the lower bounds of the uncertainty relations presented in table 1. By definition both $\Delta A > 0$ and $\Delta B > 0$ are positive quantities. The question is whether or not they might become arbitrarily small. If not then they are restricted from below as $\Delta A \geqslant A_{\min} > 0$ and $\Delta B \geqslant B_{\min} > 0$. $A_{\min}$ and $B_{\min}$ are the smallest values possible for A and B. This is a very fundamental restriction [41] which, however, seems to arise naturally in the framework of quantum gravity. This theory should merge general relativity and quantum theory and can be traced back to some remarks given by Einstein [42]. However, the theory itself is far from being complete nor is there any consensus of how it should look like. But it was shown in many different ways from first principles that merging quantum theory and general relativity will lead to a minimal length $x_0 > 0$, see e.g. [43–57]. The consequence is that a distance x with $x < x_0$ does not have any meaning and therefore is not accessible. As a result we always have $x \geqslant x_0 > 0$.

As was worked out especially by Kempf and co-workers [58–60] and some others [51, 52] by accepting x_0 as a minimal accessible length will result in a generalized or gravitational uncertainty principle (GUP) which might be formulated as an extension of the Heisenberg uncertainty principle:

$$\Delta x \geqslant \frac{\hbar}{2\Delta p}(1 + \beta(\Delta p)^2) \tag{2}$$

Table 2. The Planck units [70–75]. The numerical values are based on the CODATA recommended values of the fundamental physical constants [77]. α is the fine structure constant. Note that some authors suggest to use corrected Planck units which are deduced by substituting $4G$ for G [41].

Constant	Definition	Value
Planck length ℓ_P	$(\hbar G/c^3)^{1/2}$	$1.616\,20 \cdot 10^{-35}$ m
Planck time t_P	$(\hbar G/c^5)^{1/2}$	$5.391\,06 \cdot 10^{-44}$ s
Planck mass m_P	$(\hbar c/G)^{1/2}$	$2.176\,51 \cdot 10^{-8}$ kg
Planck temperature T_P	$(\hbar c^5/G)^{1/2}/k_B$	$1.416\,83 \cdot 10^{32}$ K
Planck energy E_P	$(\hbar c^5/G)^{1/2}$	$1.956\,15 \cdot 10^{9}$ J
Planck density ϱ_P	$c^5/(\hbar G^2)$	$5.155\,56 \cdot 10^{96}$ kg/m^3
Planck charge q_P	$e/\sqrt{\alpha}$	$1.875\,55 \cdot 10^{-18}$ C

From the GUP (2) a minimal uncertainty $\Delta x_0 = \hbar\sqrt{\beta}$ with $\beta > 0$ was obtained. It has not only been shown that the existence of a lower bound Δx_0 has an impact on the HUP but also on quantum mechanics in general, and the properties of atoms and molecules and their interactions in particular [58–68]. Needless to say that it also plays a role in astronomy and high energy physics [63, 69].

2.3. The Planck units

It is often assumed but not proved that the lower bounds x_0 and Δx_0 are given by the Planck length ℓ_P. We therefore set $\Delta x_0 = x_0 = \ell_P$. The Planck units were originally introduced by Max Planck in 1900 [70] in order to create an unbiased system of units of the fundamental physical dimensions time, mass, and length. In his opinion this system should be unique and valid throughout the whole universe. The Planck units can be obtained by a dimensionful correct combination of the speed of light in vacuum c, the Planck constant h, the Boltzmann constant k_B, and the Newtonian constant of gravitation G. In recent years the originally proposed units have been augmented by temperature, density, and other properties [71–75]. A small collection of these units is given in table 2. It should be noted that the Planck units sometimes are defined with other numerical constants [41]. This is a result of two different definitions of the Planck force either as $F_P = F_{max} = c^4/G = 1.210\,34 \cdot 10^{44}$ N [71] or as $F_P = \mathrm{F}_{max}/4 = c^4/(4G)$ [41, 76].

In the following we use the Planck units of time and length as a natural lower bound, the Planck units of density and temperature as a natural upper bound for these properties. According to Gibson [71] the Planck unit of any product $C = AB$ might be obtained by a dimensional correct combination of the corresponding Planck units of A and B. Combinations with ℓ_P, t_P, T_P, and ϱ_P are considered to be a natural bound to any property examined in this work. The lower limit of the HUP will then result in $\Delta p \Delta x = m_P c \ell_P = \hbar$. We now apply this simple combination scheme to all uncertainty relations listed in table 1. In addition to table 2 the Planck units of entropy $S_P = k_B$ and linear momentum $p_P = m_P c$ are also used [71, 75]. In the case of the indeterminacy relations (3)—(5), (7), (8), (10), (11), (13), and (14) the exact lower bounds are recovered. For the remaining five relations only a factor of (1/2) or $(1/4\pi)$ is missing. Obviously our heuristic scheme works quite well and the simple use of Planck units enables us to recover the lower bounds of the indeterminacy relations quite easily.

3. Application of the inequalities and indeterminacy relations

3.1. Viscosity and entropy—the Kovtun-Son-Starinets conjecture

We start with the Kovtun-Son-Starinets conjecture already mentioned in the introduction. In (1) we have seen, that the fundamental lower bound $\kappa/s \geqslant \kappa_0 = \hbar/(4\pi k_B) \approx 6.08 \cdot 10^{-13}$ K s exists. The lower bound of this inequality can be recovered easily with our afore mentioned treatment with the Planck units. We write the corresponding Planck ratio κ_P as

$$\kappa_P = \frac{\eta_P}{s_P} = \frac{\nu_P \varrho_P}{S_P/V_P} = T_P t_P = \frac{\hbar}{k_B}. \tag{3}$$

Here $\nu_P = (\hbar G/c)^{1/2}$ is the Planck unit of the kinematic viscosity [71] and $V_P = \ell_P^3$ the Planck volume. The result is $\kappa_0 = \kappa_P/(4\pi)$. Beside the factor $1/(4\pi)$ the lower bound of the KSS conjecture κ_0 is recovered with this simple procedure. This is in full accord with the observations summarized in table 1. Beside a numerical factor which is close to one the lower bounds of the corresponding inequalities can be obtained from the Planck units.

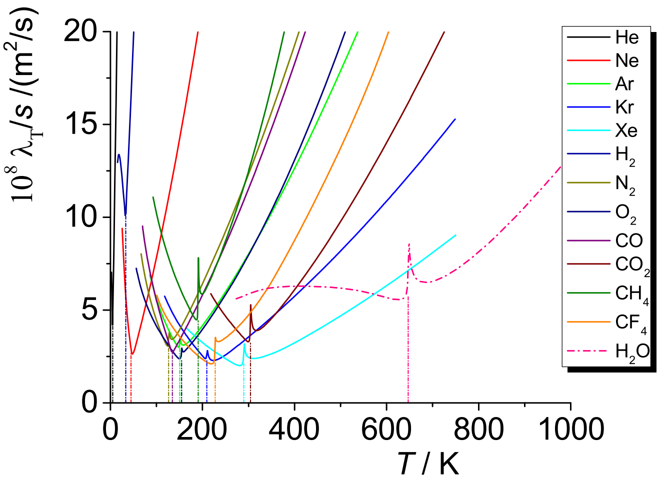

Figure 1. Critical isobars of the ratio λ_T/s obtained from tabulated thermophysical data [83]. The vertical lines correspond to the critical temperatures [83].

3.2. Thermal conductivity, diffusion, and entropy

Shear viscosity η and density of entropy $s = S/V$ are transport and equilibrium properties of fundamental importance in e.g. physical chemistry and chemical engineering. The question is if there are other thermophysical properties and combinations thereof which obey an inequality like (1). We consider the two transport coefficients thermal conductivity λ_T and self diffusion D_{11} in relation to the density of entropy s. It was discussed by Rosenfeld [78, 79] and others [80–82] that a possible relation between s and the transport coefficients η, λ_T and D_{11}, respectively, might exist. Therefore, its reasonable to study the following three ratios. According to our proposal we obtain the lower bounds from the corresponding Planck units.

$$\frac{\lambda_T}{s} \geqslant \frac{\lambda_{TP}}{s_P} = \frac{E_P/(t_P \ell_P T_P)}{S_P/V_P} = \frac{\ell_P^2}{t_P} = \left(\frac{\hbar G}{c}\right)^{1/2} \approx 4.85 \cdot 10^{-27}\ \mathrm{m^2\ s^{-1}} \tag{4}$$

$$\frac{D_{11}}{s} \geqslant \frac{D_{11P}}{s_P} = \frac{\ell_P^2/t_P}{S_P/V_P} = \frac{T_P t_P}{\varrho_P} = \frac{G^2\hbar^2}{k_B c^5} \approx 1.48 \cdot 10^{-108}\ \mathrm{K\ s\ m^3\ kg^{-1}} \tag{5}$$

$$\frac{\varrho\ D_{11}}{s} \geqslant \frac{\varrho_P D_{11P}}{s_P} = \frac{(m_P/V_P)(\ell_P^2/t_P)}{S_P/V_P} = \frac{m_P \ell_P^2}{S_P t_P} = \frac{\hbar}{k_B} \approx 7.64 \cdot 10^{-12}\ \mathrm{K\ s} \tag{6}$$

In the case of the ratio λ_T/s thermophysical data can be found in [83] for the same gases as in the case of η/s [11]. The results are shown in figure 1. In all cases an experimentally obtained minimum value of λ_T/s can be found in the vicinity of the critical temperature. However, the temperature dependence of the critical isobars is more complicated compared to the behaviour of κ. Nevertheless, the minimum values can be found in the range of $(\lambda_T/s)_{\min} \approx (2\ \ldots\ 10) \cdot 10^{-8}\ \mathrm{m^2/s}$ which is twenty orders of magnitude above the proposed theoretical limit λ_{TP}/s_P.

Experimentally determined self-diffusion coefficients D_{11} of pure substances over wide ranges of temperature and pressure are much harder to find [84]. Therefore, we concentrate ourselves to a region in the vicinity of the critical point where some experimental data can be found [84–88]. We just mention the result for methane given by Oosting and Trappeniers [86]. They report a value in the range of $D_{11} \approx (58.5 \ldots 67) \cdot 10^{-9}\ \mathrm{m^2\ s^{-1}}$ in the critical region. This gives an experimental value of $D_{11}/s \approx (4.2\ \ldots\ 4.8) \cdot 10^{-14}\ \mathrm{K\ s\ m^3/kg} \gg D_{11P}/s_P$. This is even 94 orders of magnitude above the Planck limit given in (5). Although the limits proposed in (4) and (5) strictly hold, at a first glance these two inequalities might be regarded as irrelevant. This is in strong contrast to the limit $\kappa \geqslant \kappa_0$ proposed in (1) which seems to be of relevance even in typical thermophysical applications.

Often, diffusion coefficients are tabulated as ϱD_{11}, ϱ being the mass-density of the system [89]. This leads to the inequality (6). Note that the lower bound $\varrho_P D_{11P}/s_P$ is given by the same Planck limit $\hbar/k_B$ as in the case of η/s. We use experimentally determined self-diffusion coefficients D_{11} [85–88, 90] as well as tabulated densities and entropies [83] at the critical point and obtain $\varrho D_{11}/s$ in the sequence neon, argon, krypton, xenon, hydrogen, and methane as $2.0 \cdot 10^{-12}$, $1.1 \cdot 10^{-11}$, $1.64 \cdot 10^{-11}$, $2.5 \cdot 10^{-11}$, $2.54 \cdot 10^{-12}$, and $7.94 \cdot 10^{-12}$ (all in Ks), respectively. We notice that neon and hydrogen do not obey the inequality (6). In order to settle this shortcoming we might suppose that a factor of $1/(4\pi)$ is missing on its right-hand-side. This fact was already noticed for κ where we found $\kappa_0 = \kappa_P/(4\pi)$. The missing term $1/(4\pi)$ can also be deduced from the temperature-length inequality given in table 1. Indeed, $\hbar/(4\pi k_\mathrm{B})$ is a lower bound to the afore mentioned experimental results for $\varrho D_{11}/s$. We conclude that beside inequality (1) also inequality (6) seems to be of relevance for ordinary fluids. This, however, is a somewhat expected result provided that diffusion coefficients and viscosities show no anomalies at T_C [91]. Indeed we notice that the experimentally determined ratio $(\varrho_\mathrm{m} D_{11}/s)_{\mathrm{T_C}}$ correlates very well with $(\eta/s)_{\min}$ for these fluids at the critical temperature $T \approx T_C$ [11]. In particular we obtain $(\eta/s)_{\min}/(\varrho_\mathrm{m} D_{11}/s)_{\mathrm{T_C}}$ in the range between 2.0 and 2.15 for argon, krypton, xenon and hydrogen, 5.1 in the case of neon and 1.55 for methane. These ratios at least have the same order of magnitude for the six gases and $(\eta/s)_{\min}$ approximately scales like $(\varrho_\mathrm{m} D_{11}/s)_{\mathrm{T_C}}$.

3.3. The Wiedemann–Franz law

The Wiedemann–Franz law states that for metals the ratio of the thermal and electrical conductivity, λ_T/σ, is directly proportional to the temperature T, see e.g. [92, 93].

$$\frac{\lambda_\mathrm{T}}{\sigma} = LT \tag{7}$$

The proportionality constant L is known as the Lorenz number. It is approximately constant for a variety of metals and alloys. Beside various erroneous attempts L was first calculated correctly by Sommerfeld [94] using Fermi–Dirac statistics:

$$L_0 = \lim_{T\to 0} L = \frac{\pi^2}{3}\left(\frac{k_\mathrm{B}}{e}\right)^2 \approx 2.44 \cdot 10^{-8}(\mathrm{V/K})^2 \tag{8}$$

In finite temperature experiments $L \approx L_0$ is observed. Relation (7) does not only hold to a very good approximation for ordinary metals, alloys and degenerate semiconductors [92, 93, 95], but also for some Fermi liquids [96] in general. It has been shown by a number of theoretical and experimental observations that (8) is strictly valid for metals and alloys [97–100] as well as Fermi liquids including heavy fermionic systems like $\mathrm{CeAl_3}$, $\mathrm{CeCu_6}$, $\mathrm{UPt_3}$ [101–103]. It also holds strictly for some quantum critical metals [96] and some non-Fermi liquids [104]. However it complety fails for superconductors due to Cooper-pairing of the electrons [102, 105]. As outlined before we construct a Lorenz number L_P by using the Planck units for thermal and electrical conductivity and the temperature. We set L_P as a lower limit to L and restrict ourselves to electrons in metals which behave like a Fermi liquid and obtain

$$L = \frac{\lambda_\mathrm{T}}{\sigma T} \geqslant L_\mathrm{P} = \frac{\lambda_\mathrm{TP}}{\sigma_\mathrm{P} T_\mathrm{P}} = \frac{E_\mathrm{P}/(t_\mathrm{P}\ell_\mathrm{P} T_\mathrm{P})}{q_\mathrm{P}^2/(t_\mathrm{P} E_\mathrm{P} \ell_\mathrm{P}) T_\mathrm{P}} = \frac{\alpha E_\mathrm{P}^2}{T_\mathrm{P}^2 e^2} = \alpha\left(\frac{k_\mathrm{B}}{e}\right)^2 \approx 5.42 \cdot 10^{-11}(\mathrm{V/K})^2 \tag{9}$$

First we note that $L_0 \geqslant L_\mathrm{P}$ holds. This means that our proposed lower limit L_P is in good accordance with the rigorously calculated Sommerfeld limit L_0. Second, experimental results at various temperatures show that the prediction $L \geqslant L_\mathrm{P}$ is fulfilled for a large number of metals, alloys and degenerate semiconductors [93, 95]. In contrast to the experimental results compiled in [93, 95] in the case of silver Gloos *et al* [106] observed a severe deviation from the Wiedemann–Franz law between temperatures of 2 K and 9 K. In particular they measured $L_\mathrm{exp} = (0.1 \ldots 2) \cdot 10^{-8}$ $(\mathrm{V/K})^2 < L_0$. According to Gloos *et al* [106] the unexpected wide range of experimental results is based on the different purities and treatments of the Ag samples. Nevertheless, even in this exceptional case $L_\mathrm{exp} > L_\mathrm{P}$ is observed. Hence we can conclude, that the Planck-limit L_P of the Lorenz number holds for fermionic systems.

3.4. Time and temperature

By using some heuristic arguments de Sabbata and Sivaram [23, 24] obtained the indeterminacy relation $\Delta T \Delta t \geqslant (\Delta T \Delta t)_0 = \hbar/k_\mathrm{B} \approx 7.64 \cdot 10^{-12}$ Ks by introducing torsion in general relativity. Assuming again that the uncertainties ΔT and Δt are limited by the corresponding Planck units T_P and t_P, we obtain exactly the same bound $\Delta T \Delta t \geqslant T_\mathrm{P} t_\mathrm{P} = (\Delta T \Delta t)_0 = \hbar/k_\mathrm{B}$ as was given in [23, 24]. This relation was tested in a typical laboratory experiment by Gillies and Allison [29, 30]. These authors analyze the laser-induced fluorescence decay of nanoparticles of YAG:Ce. They obtained an experimentally determined minimal value in the range of $(\Delta T \Delta t)_{\min} = (4.5 \cdot 10^{-11} \ldots 2.0 \cdot 10^{-9})$ Ks $\geqslant (\Delta T \Delta t)_0 \approx 7.64 \cdot 10^{-12}$ Ks in full accord with the lower bound proposed in [23, 24]. Interestingly $(\Delta T \Delta t)_{\min}$ is only (6...262) times above the theoretical limit $(\Delta T \Delta t)_0$, which is obtained from theoretical considerations in the field of general relativity. Again it should be stressed that the lower bound which is approached by experiment has the dimension temperature times time, exactly the same as for η/s and $\varrho D_{11}/s$.

A quite similar relation was developed by Hod [107] from quantum information theory and thermodynamics. He obtained the inequality $T\tau \geqslant \hbar/(\pi k_\mathrm{B})$. τ is the relaxation time of a perturbed thermodynamic system. This bound is called 'TTT' (time times temperature) [108] and it may be used as a quantitative way to explain the third law of thermodynamics [107]. A similar inequality was already conjectured by Sachdev [109] and the bound on the relaxation time is applied to e.g. quantum critical phenomena [110] as well as in the framework of incoherent metallic transport [111].

3.5. Photon and graviton lifetimes

In this last example we dare to address two fundamental questions in physics. What is the mass of the photon and the graviton and do they have an infinite lifetime? We propose that the uncertainty relation $\Delta M \Delta t \geqslant \hbar/c^2$ given by Landau and Peierls [16] and Aharonov and Reznik [28] might be of relevance in view of the mass and lifetime limits, ΔM and Δt, of the photon [112, 113] and graviton [112, 114]. First we mention that the same lower limit can be obtained from the product of the Plack units which also gives $\Delta M \Delta t \geqslant m_P t_P = \hbar/c^2 \approx 1.17 \cdot 10^{-51}$ kg s. We test this inequality bearing in mind that the lower limits of the mass ΔM [112] and especially of the lifetime Δt [113, 114] of the photon and graviton are still to some extend speculative and may both vary by several orders of magnitude. By using the lowest limits given in the literature we obtain for the photon $\Delta M \Delta t \approx 3.15 \cdot 10^{-37}$ kg s and in the case of the graviton $\Delta M \Delta t \approx 1.00 \cdot 10^{-38}$ kg s. We see that the proposed bound on $\Delta M \Delta t \geqslant \hbar/c^2$ holds very well.

4. Discussion and conclusion

We have seen in table 1 that our simple heuristic procedure recovers the limits of the indeterminacy relations given in the literature. However, sometimes a factor of $1/2$ or $1/(4\pi)$ is missing. This might be a drawback. But e.g. in the case of the uncertainty relation $\Delta p \Delta t \geqslant \beta\hbar/c$ even in extensive calculations the rhs is given with $\beta = 1$ [16] or $\beta = 1/2$ [34]. We have formulated new bounds for the ratio of important thermophysical properties, namely $D_{11}/s \geqslant G^2\hbar^2/(k_B c^5)$, $\lambda_T/s \geqslant (\hbar G/c)^{1/2}$, and $\varrho\ D_{11}/s \geqslant \hbar/k_B$, respectively. In the case of the first two inequalities experimental results are several orders of magnitude larger than the bounds given on the rhs Experimental results for $\varrho D_{11}/s$ are close to or even below the proposed limit of $\hbar/k_B$. However, the inequality holds if a factor of $1/(4\pi)$ is introduced in the rhs which then reads $\hbar/(4\pi k_B)$. The question in general is if this bound is of similar importance as the KSS-bound discussed in section 3.1. We also have tackled the Wiedemann–Franz law. In the case of fermionic systems we have proposed the lower limit of $\lambda_T/(\sigma T) \geqslant \alpha(k_B/e)^2$. This inequality holds well, the theoretical result of Sommerfeld as well as experiments are roughly 200 times above this limit. The lower bounds of the inequalities contain the fundamental physical constants $\hbar$, k_B, e, c, and G. In the course of these studies we notice that the bounds are approached quite closely by ordinary (not exotic high energy) matter if only the first three of these fundamental constants occur in the corresponding Planck limit. We have seen this in the cases of η/s, $\varrho\ D_{11}/s$, $\lambda_T/(\sigma T)$, and $\Delta t \Delta T$, but not for D_{11}/s and λ_T/s where the Newtonian constant G and the speed of light in vacuum c enter the corresponding lower bounds. G is the characteristic fundamental constant in general relativity and c in special relativity. On account of these observations we can conclude that effets from general relativity do not play a significant role for ordinary bulk matter behaviour. If in contrast to this the lower bound of an inequality is given by the Planck limit $\hbar/k_B$ then this limit might be of relevance in every day physico-chemical investigations. This consequence should be tested for a variety of other physico-chemical properties.

5. Conflict of interest

The authors declare that they have no conflict of interest.

6. List of symbols

Symbol	Meaning	Symbol	Meaning
A	Observable	α	Fine structure constant
c	speed of light in vacuum	η	shear viscosity
D_{11}	Coefficient of self diffusion	κ	Ratio of shear viscosity to density of entropy
e	Elementary charge	λ_T	Thermal conductivity
E	Energy	μ	Chemical potential
E_P	Planck energy	ν	Kinematic viscosity
F	Helmholtz energy	Π	Pressure
G	Newtonian constant of gravity	ϱ_P	Planck density
h	Planck's constant	σ	Electrical conductivity
$\hbar = h/2\pi$	reduced Planck's constant		

(Continued.)

Symbol	Meaning	Symbol	Meaning
J	Angular momentum		
k_B	Boltzmann's constant		
L	Lorentz number		
ℓ_P	Planck length		
m	Mass		
m_P	Planck mass		
N	particle number		
p	momentum		
q_P	Planck charge		
S	Entropy		
$s = S/V$	density of entropy		
$\dot{S}$	Entropy production rate		
t	Time		
T	Temperature		
T_C	Critical temperature		
t_P	Planck time		
T_P	Planck temperature		
V	Volume		
V_P	Planck volume		
x	position		

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References

[1] Kovtun P K, Son D T and Starinets A O 2005 Viscosity in strongly interacting quantum field theories from black hole physics *Phys. Rev. Lett.* **94** 111601

[2] Venugopalan R 2008 From glasma to quark-gluon plasma in heavy-ion collisions *J. Phys. G: Nucl. Part. Phys.* **35** 104003

[3] Zhou C L, Ma Y G, Fang D Q, Li S X and Zhang G Q 2012 Ratio of shear viscosity to entropy density in multifragmentation of Au + Au *EPL* **98** 66003

[4] Schäfer T and Teaney D 2009 Nearly perfect fluidity: from cold atomic gases to hot quark gluon plasmas *Rep. Prog. Phys.* **72** 126001

[5] Brustein R and Medved A J M 2010 Proof of a universal lower bound on the shear viscosity to entropy density ratio *Phys. Lett.* B **691** 87–90

[6] Cao C, Elliott E, Joseph J, Wu J, Petricka H, Schäfer T and Thomas J E 2011 Universal quantum viscosity in a unitary fermi gas *Science* **331** 58–61

[7] Cremonini S 2011 The shear viscosity to entropy ratio: a status report *Mod. Phys. Lett.* B **25** 1867–88

[8] Adams A, Carr L D, Schäfer T, Steinberg P and Thomas J E 2012 Strongly correlated quantum fluids: ultracold quantum gases, quantum chromodynamic plasmas and holographic duality *New J. Phys.* **14** 115009

[9] Schäfer T 2014 Fluid dynamics and viscosity in strongly correlated fluids *Annu. Rev. Nucl. Part. Sci.* **64** 125–48

[10] Hartnoll S A, Ramirez D M and Santos J E 2016 Entropy production, viscosity bounds and bumpy black holes *J. High Energy Phys.* JHEP03(2016)170

[11] Hohm U 2014 On the ratio of the shear viscosity to the density of entropy of the rare gases and H_2, N_2, CH_4, and CF_4 *Chem. Phys.* **444** 39–42

[12] Fang D Q, Ma Y G and Zhou C L 2014 Shear viscosity of hot nuclear matter by the mean free path method *Phys. Rev.* C **89** 047601

[13] Rameau J D, Reber T J, Yang H-B, Akhanjee S, Gu G D and Johnson P D 2014 Nearly perfect fluidity in a high-temperature superconductor *Phys. Rev.* B **90** 134509

[14] Heisenberg W 1927 Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik *Z. Phys.* **43** 172–98

[15] Busch P, Lahti P and Werner R F 2014 Quantum root-mean-square error and measurement uncertainty relations *Rev. Mod. Phys.* **86** 1261–81

[16] Landau L and Peierls R 1931 Erweiterung des Unbestimmtheitsprinzips für die relativistische Quantentheorie *Z. Phys.* **69** 56–69

[17] Mandelbrot B 1956 An outline of a purely phenomenological theory of statistical thermodynamics: I. canonical ensembles *IRE Trans. Inf. Theory* **2** 190–203

[18] Rosenfeld L 1961 Questions of irreversibility and ergodicity *Proc. Enrico Fermi School of Physics* 14 1–20

[19] Guth E 1962 New class of classical uncertainty relations giving uncertainty for long and certainty for short times *Phys. Rev.* **126** 1213–5

[20] Heisenberg W 1969 *Der Teil und das Ganze, Chapter 9* (München, Zürich: Piper)

[21] Lindhard J 1986 'Complementarity' between energy and temperature *The Lessons of Quantum Theory* ed J de Boer, E Dal and O Ulfbeck (Amsterdam: Elsevier Science)

[22] Schlögl F 1988 Thermodynamic uncertainty relation *J. Phys. Chem. Solids* **49** 679–83

[23] de Sabbata V and Sivaram C 1992 A minimal time and time-temperature uncertainty principle *Found. Phys. Lett.* **5** 183–9

[24] de Sabbata V 1994 The importance of spin and torsion in the early universe *Il Nuovo Cimento* **107A** 363–74

[25] Uffink J and van Lith J 1999 Thermodynamic uncertainty relations *Found. Phys.* **29** 655–92

[26] Lavenda B H 2000 Comments on 'thermodynamic uncertainty relations' by J Uffink and J van Lith *Found. Phys. Lett.* **13** 487–92

[27] Uffink J and van Lith J 2001 Thermodynamic uncertainty relations: a reply to Lavenda *Found. Phys. Lett.* **14** 187–93

[28] Aharonov Y and Reznik B 2000 'Weighing' a closed system and the time-energy uncertainty principle *Phys. Rev. Lett.* **84** 1368–70

[29] Gillies G T and Allison S W 2004 Experimental Test of a Time-Temperature Formulation of the uncertainty principle *The Gravitational Constant: Generalized Gravitational Theories and Experiments* ed V de Sabbata, G T Gillies and V N Melnikov (Dordrecht: Kluwer Academic Publishers)

[30] Gillies G T and Allison S W 2005 Experimental test of a time-temperature formulation of the uncertainty principle via nanoparticle fluorescence *Found. Phys. Lett.* **18** 65–74

[31] Zimmermann H W 2003 Particle entropies and entropy quanta: IV. the ideal gas, the second law of thermodynamics, and the p-t uncertainty relation *Z. Phys. Chem.* **217** 55–78

[32] Zimmermann H W 2006 Quantum thermodynamics of entropy production and time—an approach to irreversibility *Z. Phys. Chem.* **220** 1131–58

[33] Zimmermann H W 2011 Relation between quantum thermodynamics and classical thermodynamics *Z. Phys. Chem.* **225** 1–13

[34] Dodonov V V and Dodonov A V 2015 Energy—time and frequency—time uncertainty relations: exact inequalities *Phys. Scr.* **90** 074049

[35] Barato A C and Seifert U 2015 Thermodynamic uncertainty relation for biomolecular processes *Phys. Rev. Lett.* **114** 158101

[36] Gingrich T R, Horowitz J M, Perunov N and England J 2016 Dissipation bounds all steady-state current fluctuations *Phys. Rev. Lett.* **116** 120601

[37] Proesmans K and Van den Broeck C 2017 Discrete-time thermodynamic uncertainty relation *EPL* **119** 20001

[38] Burderi L, Di Salvo T and Iaria R 2016 Quantum clock: a critical discussion on spacetime *Phys. Rev.* D **93** 064017

[39] Viaggiu S 2016 Entropy, energy and temperature-length inequality for Friedmann universes *Int. J. Mod. Phys.* D **25** 1650033

[40] Miller H J D and Anders J 2018 Energy-temperature uncertainty relation in quantum thermodynamics *Nature Comm.* **9** 2203

[41] Schiller C 2017 Motion Mountain, Vol 6 Online at www.motionmountain.net

[42] Einstein A 1916 Näherungsweise Integration der Feldgleichungen der Gravitation *Monatsber. Königl. Preuss. Akad. Wiss. Berlin* 688–96

[43] Wheeler J A 1957 On the nature of quantum geometrodynamics *Ann. of Physics* **2** 604–14

[44] Mead C A 1964 Possible connection between gravitation and fundamental length *Phys. Rev.* **135** B849–62

[45] Treder H-J 1966 Die Quantentheorie des Gravitationsfeldes und die Plancksche Elementarlänge *Monatsber. Deutsch. Akad. Wiss. Berlin* **8** 311–8

[46] De Witt B S 1967 Quantum theory of gravity. III. applications of covariant theory *Phys. Rev.* **162** 1239–56

[47] Gallaher D F 1970 A possible model of a divergenceless nonlocal quantum field theory *Prog. Theor. Phys.* **44** 1090–1

[48] Padmanabhan T 1985 Planck length as the lower bound to all physical length scales *Gen. Rel. Grav.* **17** 215–21

[49] Amati D, Ciafaloni M and Veneziano G 1989 Can spacetime be probed below the string size? *Phys. Lett.* B **216** 41–7

[50] Konishi K, Paffuti G and Provero P 1990 Minimum physical length and the generalized uncertainty principle in string theory *Phys. Lett.* B **234** 276–84

[51] Garay L J 1995 Quantum gravity and minimum length *Int. J. Mod. Phys.* A **10** 145–65

[52] Adler R J and Santiago D I 1999 On gravity and the uncertainty principle *Mod. Phys. Lett.* A **14** 1371–81

[53] Hossenfelder S 2013 Minimal length scale scenarios for quantum gravity *Living Rev. Relativ.* **16** 2–90

[54] Calmet X, Graesser M and Hsu S D H 2004 Minimum length from quantum mechanics and classical general relativity *Phys. Rev. Lett.* **93** 211101

[55] Jou D and Mongiovi M S 2011 Duality-invariant Einstein-Planck relation and the speed of light at very short wavelengths *Phys. Rev.* D **84** 107303

[56] Jou D, Mongiovi M S and Sciacca M 2014 A duality-invariant Einstein-Planck relation and its consequences on micro black holes *Int. J. Mod. Phys.* D **23** 1450018

[57] Jou D, Sciacca M and Mongiovi M S 2015 Three duality symmetries between photons and cosmic string loops, and macro and micro black holes *Symmetry* **7** 2134–49

[58] Kempf A 1994 Uncertainty relation in quantum mechanics with quantum group symmetry *J. Math. Phys.* **35** 4483–96

[59] Kempf A, Mangano G and Mann R B 1995 Hilbert space representation of the minimal length uncertainty relation *Phys. Rev.* D **52** 1108–18

[60] Kempf A 1997 Non-pointlike particles in harmonic oscillators *J. Phys. A: Math. Gen.* **30** 2093–101

[61] Kempf A 1997 On quantum field theory with nonzero minimal uncertainties in positions and momenta *J. Math. Phys.*. **38** 1347–72

[62] Chang L N, Minic D, Okamura N and Takeuchi T 2002 Exact solution of the harmonic oscillator in arbitrary dimensions with minimal length uncertainty relations *Phys. Rev.* D **65** 125027

[63] Benczik S, Chang L N, Minic D, Okamura N, Rayyan S and Takeuchi T 2002 Short distance versus long distance physics: the classical limit of the minimal length uncertainty relation *Phys. Rev.* D **66** 026003

[64] Harbach U and Hossenfelder S 2006 The Casimir effect in the presence of a minimal length *Phys. Lett.* B **632** 379–83

[65] Panella O 2007 Casimir–Polder intermolecular forces in minimal length theories *Phys. Rev.* D **76** 045012

[66] Bouaziz D and Ferkous N 2010 Hydrogen atom in momentum space with a minimal length *Phys. Rev.* A **82** 022105

[67] Quesne C and Tkachuk V M 2010 Composite system in deformed space with minimal length *Phys. Rev.* A **81** 012106

[68] Gao D and Zhan M 2016 Constraining the generalized uncertainty principle with cold atoms *Phys. Rev.* A **94** 013607

[69] Mathew A and Nandy M K 2018 Effect of minimal length uncertainty on the mass-radius relation of white dwarfs *Ann. Phys.* **393** 184–205

[70] Planck M 1900 Ueber irreversible Strahlungsvorgänge *Ann. d. Physik* **306** 69–122

[71] Gibson C H 2004 The first turbulence and first fossil turbulence *Flow Turbul. Combust.* **72** 161–79

[72] Tegmark M, Aguirre A, Rees M J and Wilczek F 2006 Dimensionless constants, cosmology, and other dark matters *Phys. Rev.* D **73** 023505

[73] Danielewski M 2007 The Planck-Kleinert crystal *Z. Naturforsch.* **62a** 564–8

[74] Flowers J L and Petley B W 2008 Planck, units, and modern metrology *Ann. Phys. (Berlin)* **17** 101–14
[75] Buczyna J R, Unnikrishnan C S and Gillies G T 2011 Standard and derived planck quantities: selected analysis and observations *Gravitation Cosmol.* **17** 339–43
[76] Gibbons G W 2002 The maximum tension principle in general relativity *Found. Phys.* **32** 1891–901
[77] Mohr P J, Newell D B and Taylor D B 2016 CODATA recommended values of the fundamental physical constants: 2014 *J. Phys. Chem. Ref. Data* **45** 043102
[78] Rosenfeld Y 1977 Relation between the transport coefficients and the internal entropy of simple systems *Phys. Rev.* A **15** 2545–9
[79] Rosenfeld Y 1999 A quasi-universal scaling law for atomic transport in simple fluids *J. Phys.: Condens. Matter* **11** 5415–27
[80] Dzugutov M 1996 A universal scaling law for atomic diffusion in condensed matter *Nature* **381** 137–9
[81] Abramson E H 2014 Viscosity of fluid nitrogen to pressures of 10 GPa *J. Phys. Chem.* B **118** 11792–6
[82] Zhang Y-J 2014 Entropy and entropy production in some applications *Physica* A **396** 88–98
[83] Lemmon E W, McLinden M O and Friend D G 2014 *Thermophysical Properties of Fluid systems in: NIST Chemistry WebBook, NIST Standard Reference Database Number 69* ed P J Linstrom and W G Mallard (Gaithersburg MD: National Institute of Standards and Technology) 20899, http://webbook.nist.gov
[84] De S, Shapir Y and Chimowitz E H 2001 Scaling of self and Fickian diffusion coefficients in the critical region *Chem. Eng. Sci.* **56** 5003–10
[85] Hartland A and Lipsicas M 1964 Spin-diffusion measurement in hydrogen between 20 and 55°K *Phys. Rev.* **133** A665–7
[86] Oosting P H and Trappeniers N J 1971 Proton-spin-lattice relaxation and self-diffusion in methanes IV. self-diffusion in methane *Physica* **51** 418–31
[87] Hamann H, Hoheisel C and Richtering H 1972 Nuclear magnetic resonance studies and self-diffusion at critical points in fluid systems *Ber. Bunsenges. Phys. Chem.* **76** 249–53
[88] Carelli P, Modena I and Ricci F P 1973 Self-diffusion in krypton at intermediate densities *Phys. Rev.* A **7** 298–303
[89] Zarkova L and Hohm U 2002 pVT second virial coefficients B(T), viscosity η(T), and self-diffusion ρ D(T) of the gases: BF_3, CF_4 , SiF_4 , CCl_4 , $SiCl_4$, SF_6 , MoF_6, WF_6 , UF_6, $C(CH_3)_4$, and $Si(CH_3)_4$ determined by means of an isotropic temperature-dependent potential *J. Phys. Chem. Ref. Data* **31** 183–216
[90] Bewilogua L, Gladun C and Kubsch B 1971 The coefficient of self-diffusion of liquid neon *J. Low Temp. Phys.* **4** 299–303
[91] Rah K and Eu B C 1999 Relation of shear viscosity and self-diffusion coefficient for simple liquids *Phys. Rev.* E **60** 4105–16
[92] Ziman J M 1960 *Electrons and Phonons* (Oxford: Oxford University Press)
[93] Hust J G and Sparks L L 1973 *Lorenz Ratios of Technically Important Metals and Alloys, NBS Technical Note 634* (Boulder: U.S. Department of Commerce)
[94] Sommerfeld A 1928 Zur elektronentheorie der metalle auf grund der fermischen statistik *Z. Physik* **47** 1–32
[95] Kumar G S, Prasad G and Pohl R O 1993 Experimental determinations of the Lorenz number *J. Mat. Sci.* **28** 4261–72
[96] Kim K-S and Pépin C 2009 Violation of the Wiedemann–Franz law at the Kondo breakdown quantum critical point *Phys. Rev. Lett.* **102** 156404
[97] Amundsen T, Myhre A and Salter J A M 1972 The Wiedemann–Franz ratio of aluminium at liquid helium temperatures *Phil. Mag.* **25** 513–7
[98] Kus F W 1978 The electronic thermal conductivity of simple metals and alloys: lithium and aluminium *J. Phys. F: Metal Phys.* **8** 651–7
[99] Zhang Y, Ong N P, Xu Z A, Krishana K, Gagnon R and Taillefer L 2000 Determining the Wiedemann–Franz ratio from the thermal hall conductivity: application to Cu and $YBa_2Cu_3O_{6.95}$ *Phys. Rev. Lett.* **84** 2219–22
[100] Woodcraft A L 2005 Recommended values for the thermal conductivity of aluminium of different purities in the cryogenic to room temperature range, and a comparison with copper *Cryogenics* **45** 626–36
[101] Kearney M J and Butcher P N 1988 Thermal transpot in disordered systems *J. Phys. C: Solid State Phys.* **21** L265–70
[102] Hill R W, Proust C, Taillefer L, Fournier P and Greene R L 2011 Breakdown of fermi-liquid theory in a copper-oxide superconductor *Nature* **414** 711–5
[103] Li M-R and Orignac E 2002 Heat conduction and Wiedemann–Franz law in disordered Luttinger liquids *Europhys. Lett.* **60** 432–8
[104] Mahajan R, Barkeshli M and Hartnoll S A 2013 Non-fermi liquids and the Wiedemann–Franz law *Phys. Rev.* B **88** 125107
[105] Graf M J, Yip S-K and Sauls S J 1996 Electronic thermal conductivity and the Wiedemann–Franz law for unconventional superconductors *Phys. Rev.* B **53** 15145–61
[106] Gloos K, Mitschka C, Pobell F and Smeibidl P 1990 Thermal conductivity of normal and superconducting metals *Cryogenics* **30** 14–8
[107] Hod S 2007 universal bound on dynamical relaxation times and black-hole quasinormal ringing *Phys. Rev.* D **75** 064013
[108] López-Ortega A 2010 On the time times temperature bound *Int. J. Mod. Phys.* D **19** 1973–85
[109] Sachdev S 1999 *Quantum Phase Transitions* (Cambridge: Oxford University Press)
[110] Sachdev S and Müller M 2009 Quantum criticality and black holes *J. Phys. Condens. Matter* **21** 164216
[111] Hartnoll S A 2015 Theory of universal incoherent metallic transport *Nat. Phys.* **11** 54–61
[112] Goldhaber A S and Nieto M M 2010 Photon and graviton mass limits *Rev. Mod. Phys.* **82** 939–79
[113] Heeck J 2013 How stable is the photon? *Phys. Rev. Lett.* **111** 021801
[114] Modanese G 2017 Graviton decay via excitation of nonlinear vacuum fluctuations *Adv. Stud. Theor. Phys.* **11** 19–26